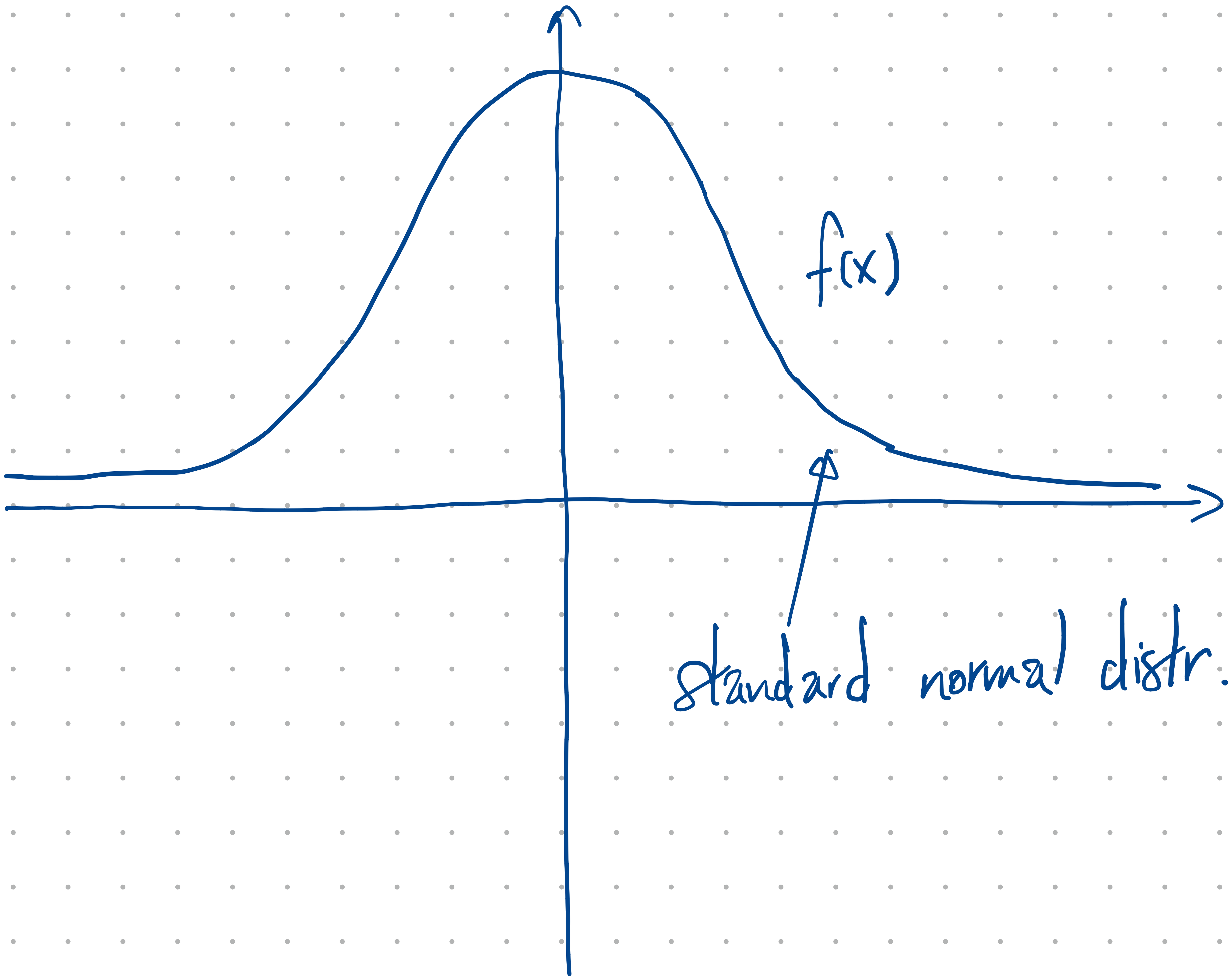
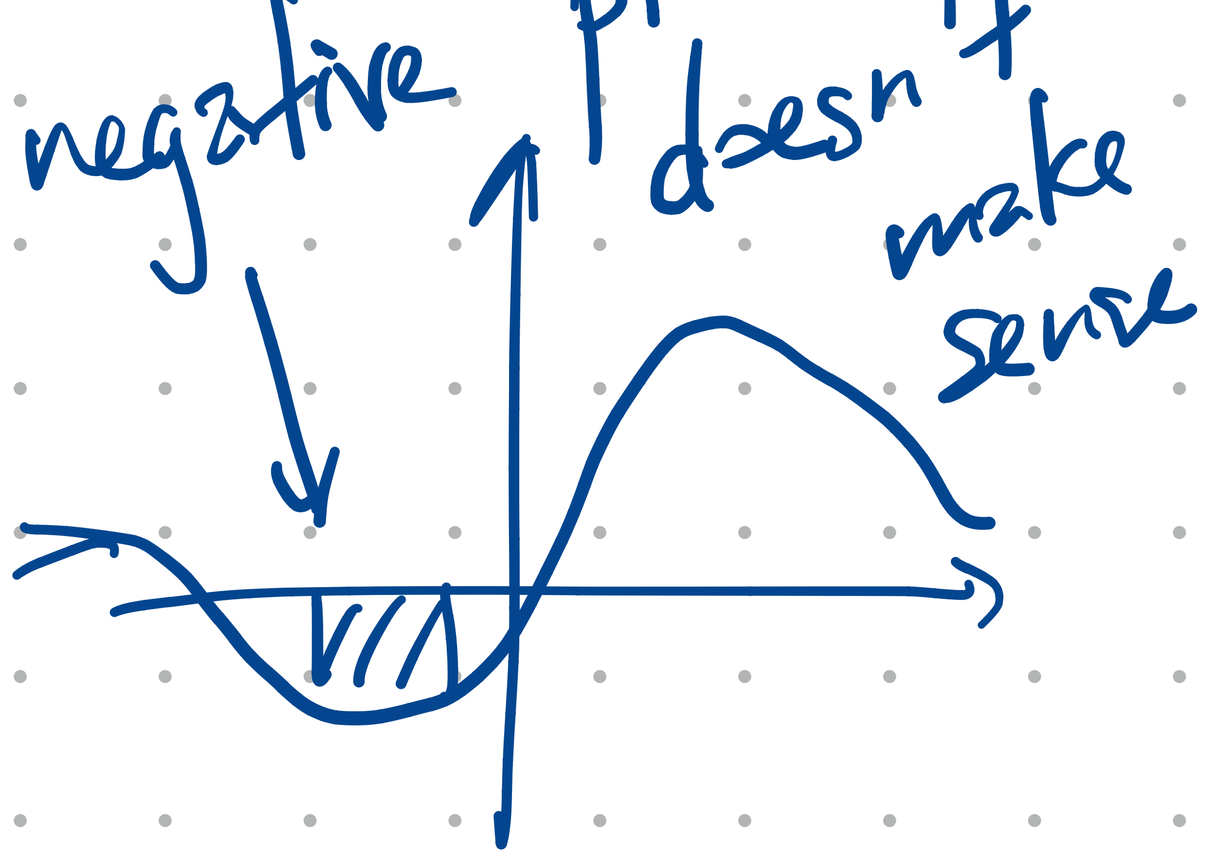


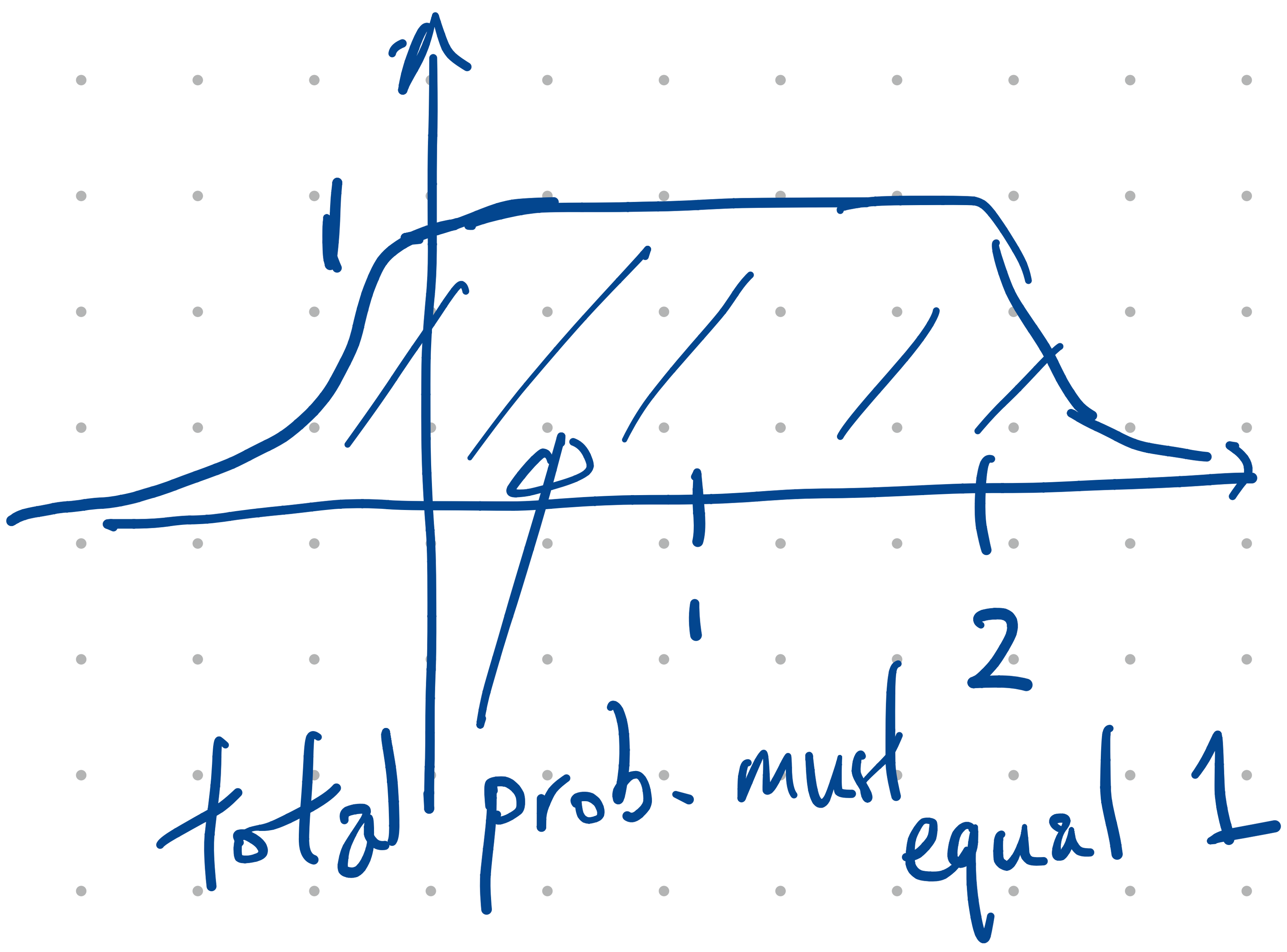
For f to be a probability density fn...

need

- $f(x) \geq 0$ always



- $\displaystyle\int_{\text{entire domain}} f(x)\,dx = 1.$



$$f(x) = C e^{-x^2/2} \quad C \text{ is some const.}$$

s.t. $$\int_{-\infty}^{\infty} C e^{-x^2/2}\, dx = 1$$

$$1/C = \int_{-\infty}^{\infty} e^{-x^2/2}\, dx$$

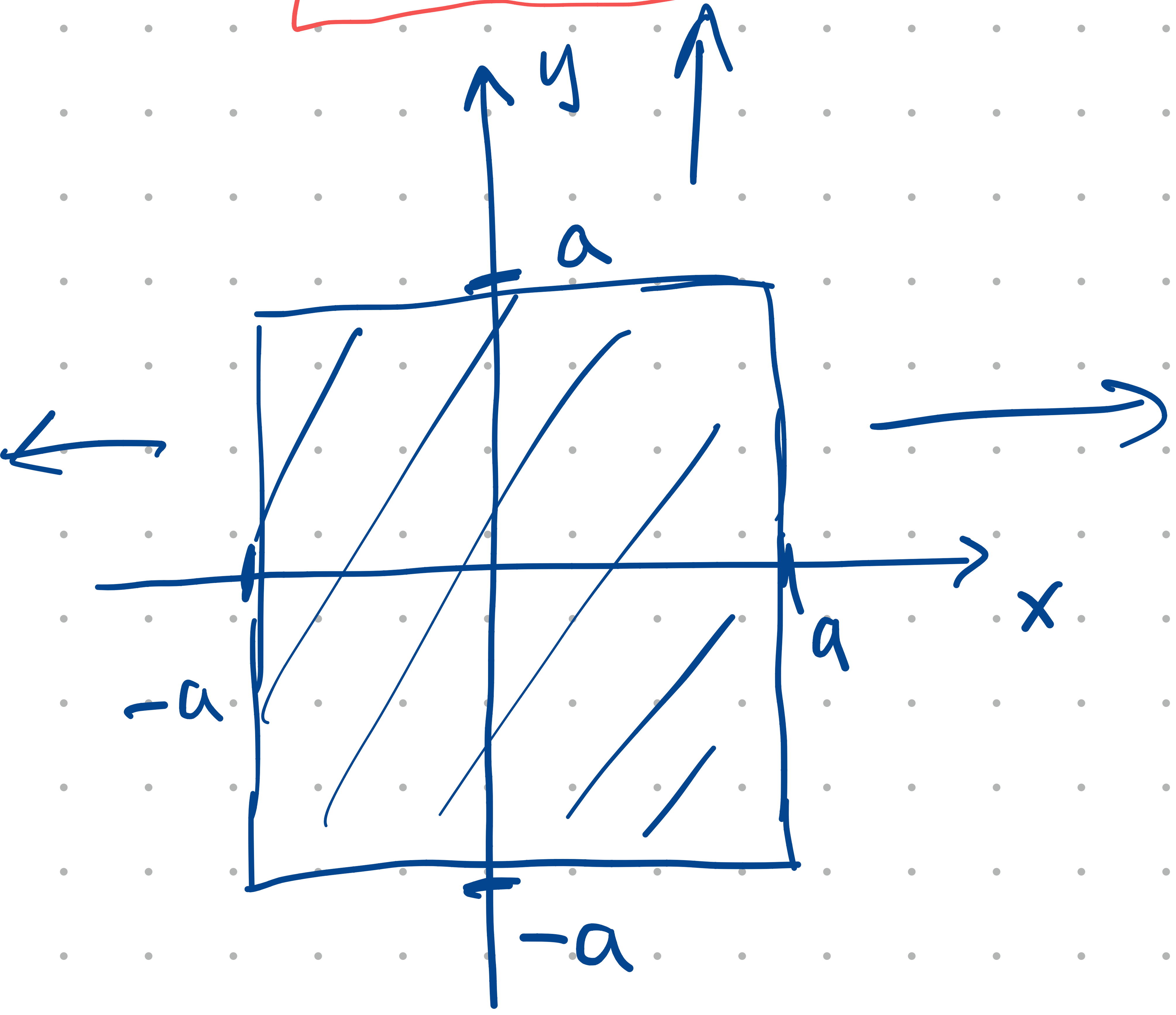
$$= \lim_{a\to\infty} \int_{-a}^{a} e^{-x^2/2}\, dx$$

$$\frac{1}{C^2} = \lim_{a\to\infty} \left(\int_{-a}^{a} e^{-x^2/2}\, dx \int_{-a}^{a} e^{-x^2/2}\, dx \right)$$

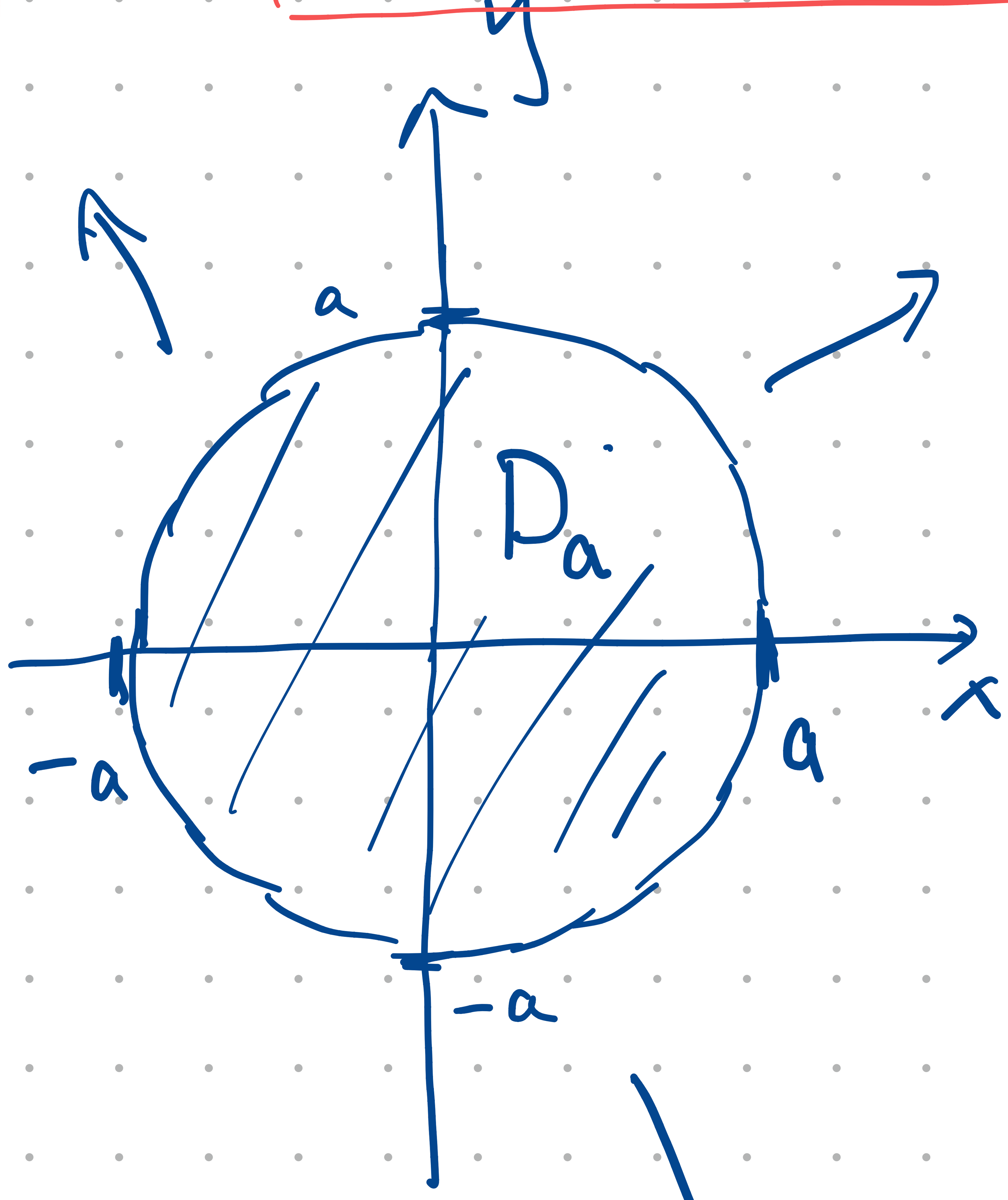
$$= \lim_{a\to\infty} \left(\int_{-a}^{a} e^{-x^2/2}\, dx \int_{-a}^{a} e^{-y^2/2}\, dy \right)$$

$$= \lim_{a\to\infty} \left(\int_{-a}^{a} \int_{-a}^{a} e^{-x^2/2}\, e^{-y^2/2}\, dx\, dy \right)$$ by Stewart eq. 15.1.11

$$= \lim_{a\to\infty} \boxed{\int_{-a}^{a}\int_{-a}^{a} e^{\frac{-x^2-y^2}{2}}\,dx\,dy} = \lim_{a\to\infty} \boxed{\iint_{D_a} e^{\frac{-x^2-y^2}{2}}\,dx\,dy}$$

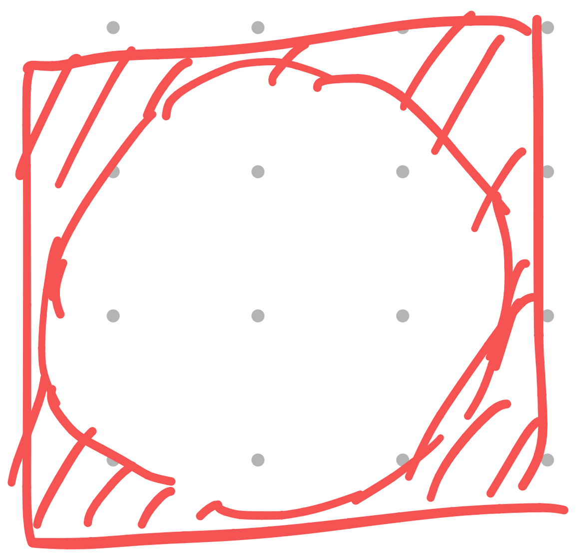


let $a \to \infty$ to cover entire plane.



let $a \to \infty$ to cover entire plane.

⚠ The boxed integrals are <u>NOT</u> the same for a fixed a.



difference is integral on this region (positive)

but the limits are the same.

$$= \lim_{a\to\infty} \int_0^{2\pi} \int_0^{a} e^{-r^2/2}\, r\, dr\, d\theta$$

$$u = -\frac{r^2}{2}$$

$$du = -r\, dr$$

$$= \lim_{a\to\infty} \int_0^{2\pi} \int_0^{-\frac{a^2}{2}} -e^{u}\, du\, d\theta$$

$$= \lim_{a\to\infty} \int_0^{2\pi} -\left(e^{-\frac{a^2}{2}} - e^{0}\right) d\theta$$

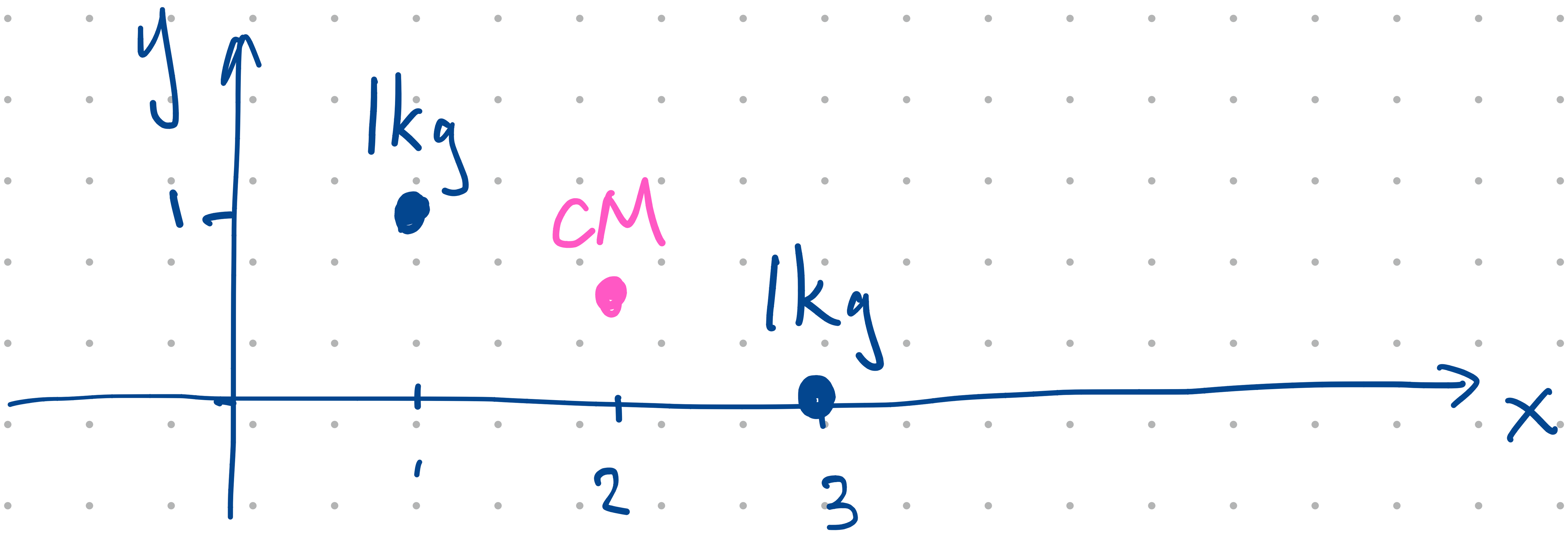
$$= \lim_{a\to\infty} 2\pi\left(1 - e^{-a^2/2}\right) = 2\pi$$

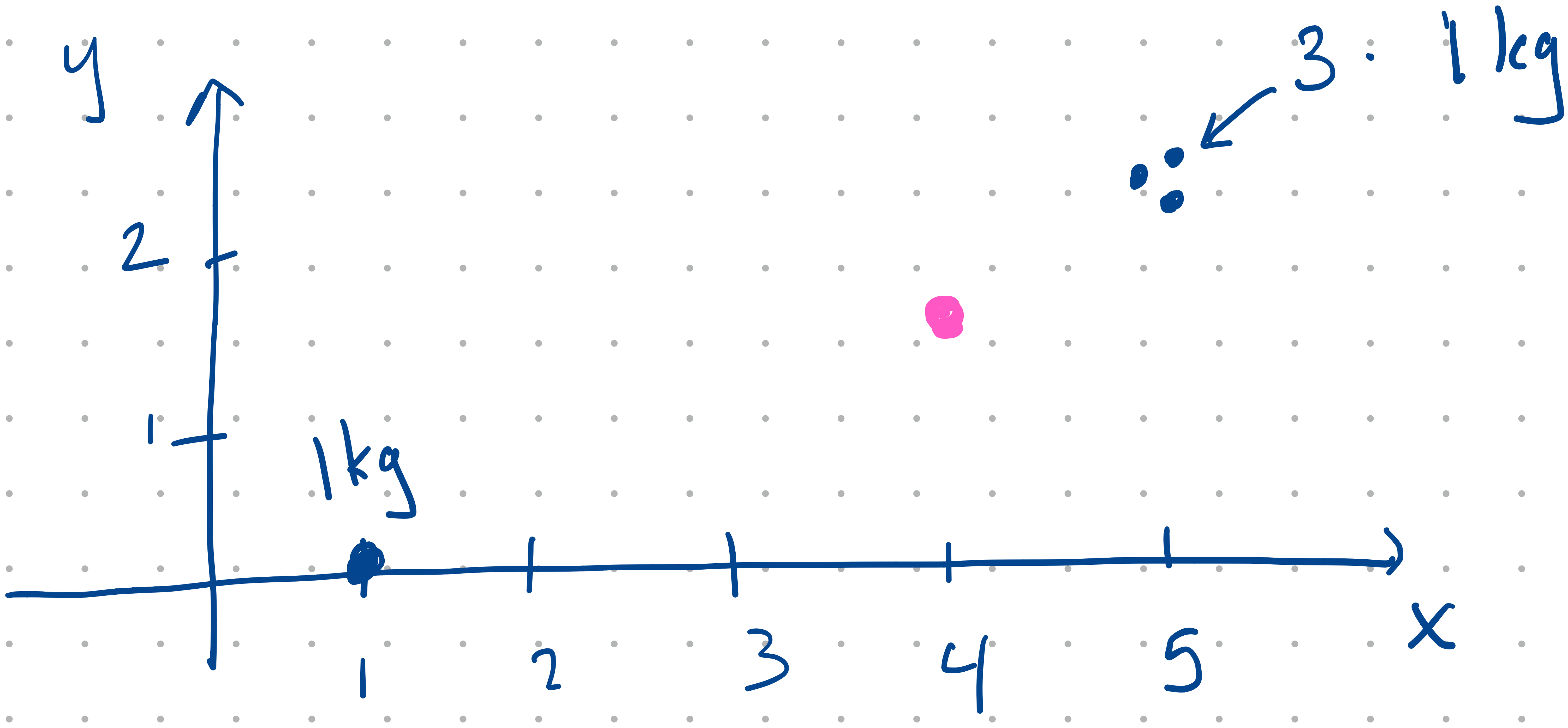
i.e. $1/c^2 = 2\pi$ so $C = \frac{1}{\sqrt{2\pi}}$

$$f(x) = \boxed{\frac{1}{\sqrt{2\pi}} e^{-x^2/2}}$$

See Exercise 15.3.40. in Stewart.

Center of mass...

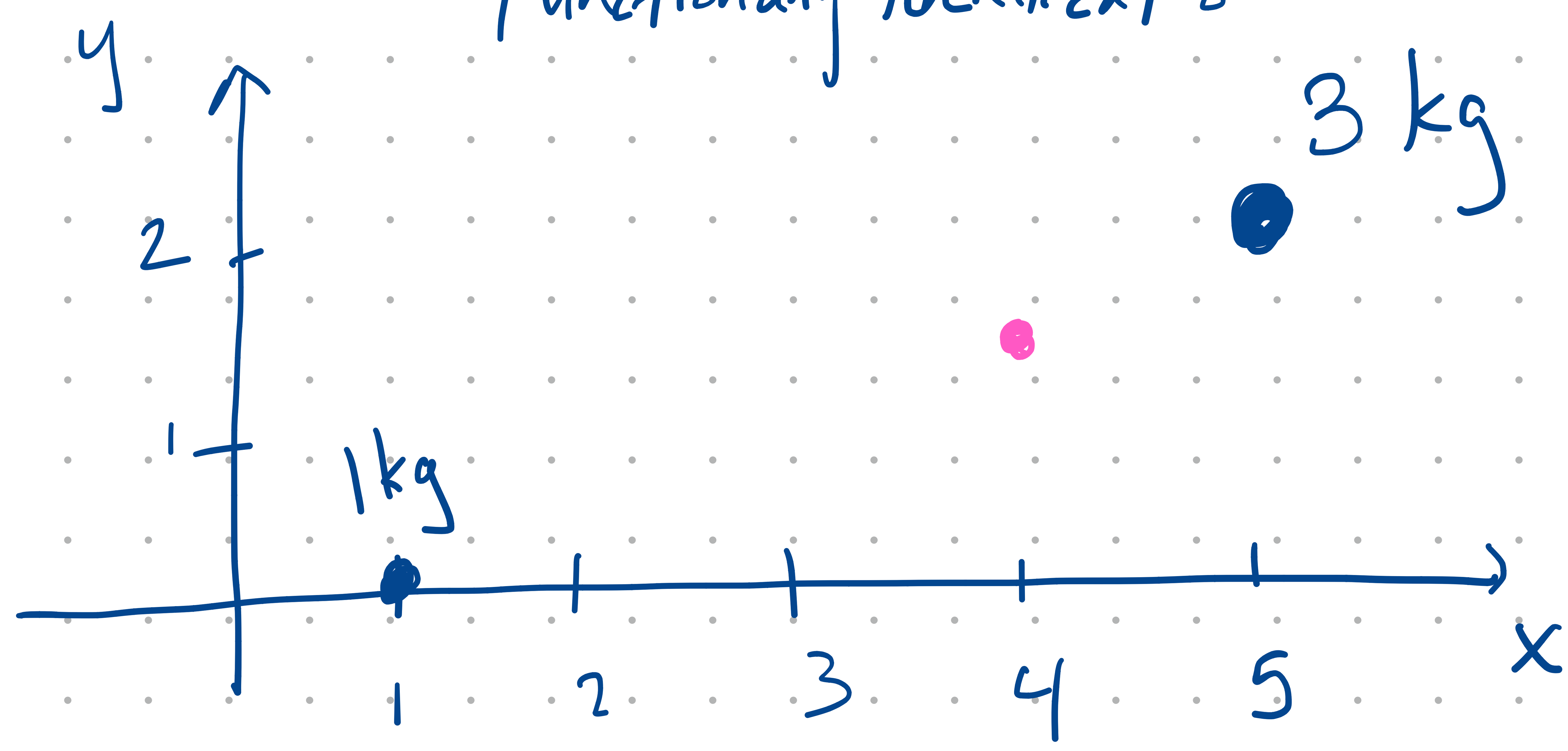




$$\frac{\langle 1,0\rangle + \langle 5,2\rangle + \langle 5,2\rangle + \langle 5,2\rangle}{4}$$

$$= \langle 4, 3/2 \rangle$$

Functionally identical situation:



$$\frac{1\langle 1,0\rangle + 3\langle 5,2\rangle}{1+3} \text{ is CM.}$$

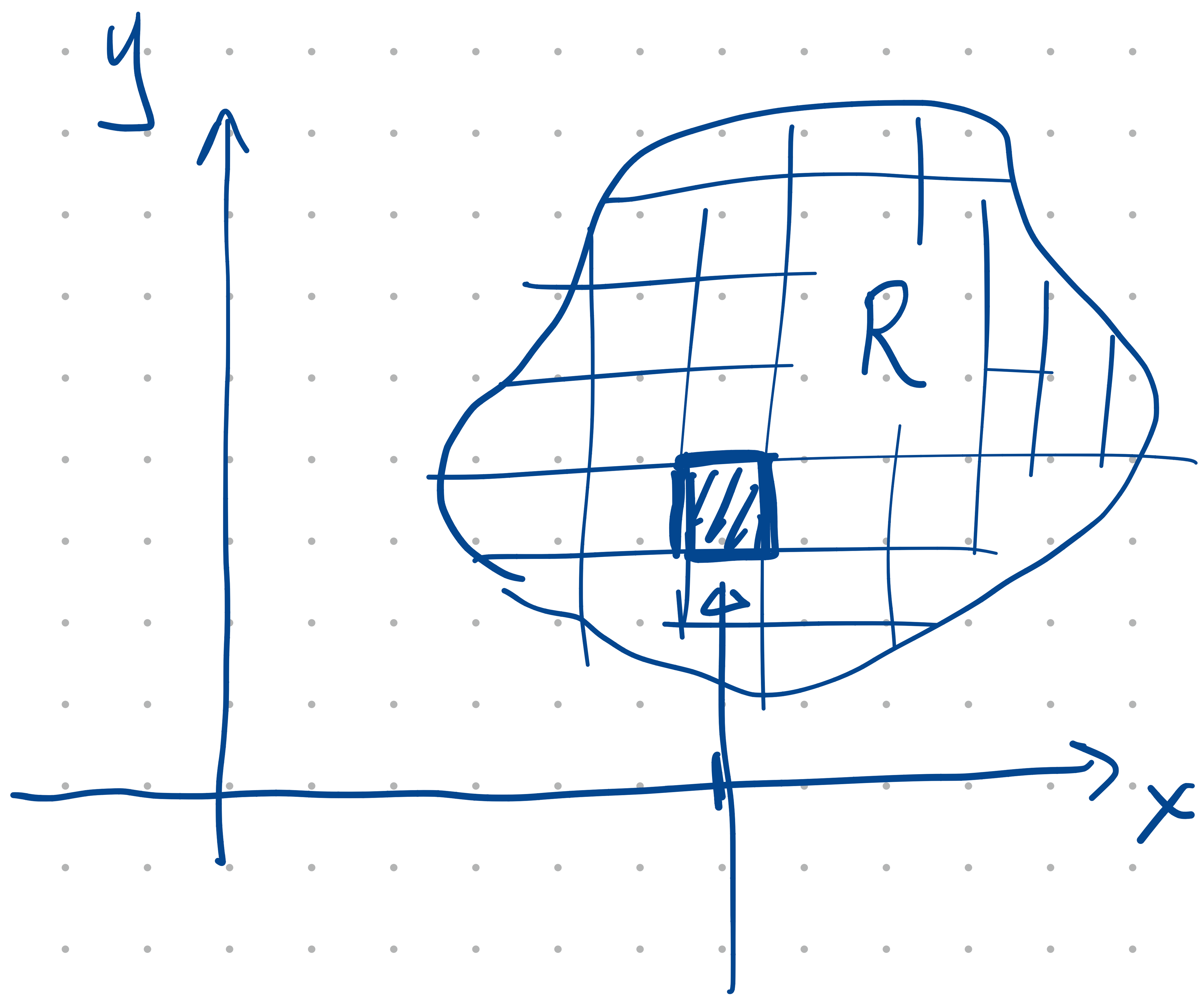
$$= \langle 4, 3/2\rangle$$

"CM is the weighted average of positions

weighted by mass"

$$x_{cm} = \frac{\sum x_i m_i}{\sum m_i} \quad \& \quad y_{cm} = \frac{\sum y_i m_i}{\sum m_i}$$

m_i is mass of object i $\langle x_i, y_i\rangle$ is location of object i



infinitesimal mass "dm" $= \sigma(x,y)\, dA$

$$x_{cm} = \frac{\iint_R x\, dm}{\iint_R dm} = \frac{\iint_R x\,\sigma(x,y)\, dA}{\iint \sigma(x,y)\, dA}$$

mass-weighted sum

total mass

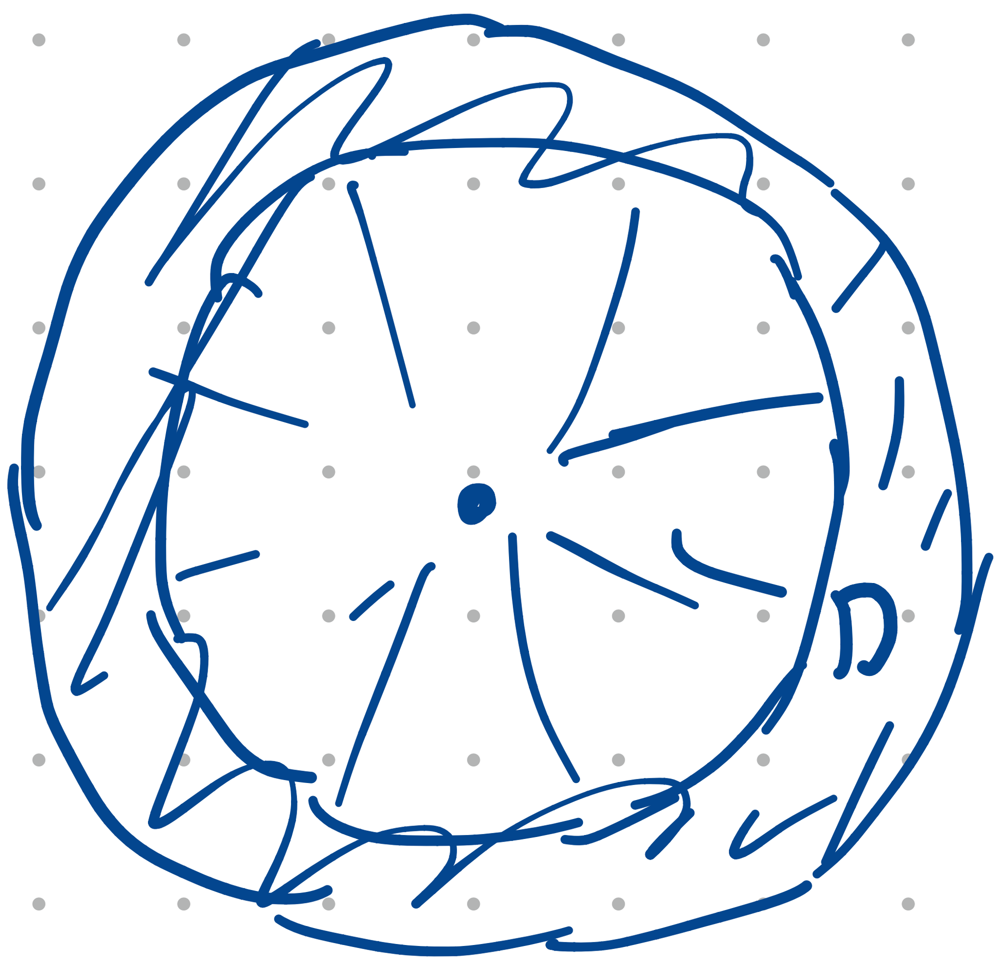
likewise for y_{cm}.

Moment of inertia = weighted sum of r^2

weighted by mass

distance to rotation axis.

"objects are harder to spin when mass is concentrated farther away from axis of rotation"



$$I = \iint_D r^2 \, dm$$

$$= \iint_D r^2 \, \sigma(x,y) \, dA$$

- if rotation axis is z-axis

$$r^2 = x^2 + y^2$$

- if ro . . . . . . — - - x-axis

$$r^2 = y^2 \quad . \quad . \quad -$$